Lecture 2: Einstein's Approach of Radiation and the Bohr's Second Postulate

The spontaneous and forced emissions are for the first time solved by Einstein (1917) throughout the introduced coefficients $A_{nn'}$ and $B_{nn' (\text{or } n'n)}$, respectively, relating the probability in unit time once electron going from a state (n) of energy E_n into another one (n') with energy $E_{n'}$ between which the unit photonic energy exists (is postulated):

$$h\upsilon = E_n - E_{n'} \quad (1.)$$

As such Einstein accepts at once both the wave quantification as well as the Bohr postulate of transitions between stationary states; while his approach will results in the Planck spectral density ρ_υ the Bohr postulate of quantum transitions follows as being with this occasion demonstrated.

Therefore, while assuming N_n and $N_{n'}$ atoms in each of the n and n' states in discussion, with $n>n'$, within the Maxwell-Boltzmann microstates,

$$N_n = \beta \exp(-\beta E_n),$$

$$N_{n'} = \beta \exp(-\beta E_{n'}),$$

the spontaneous radiant, emitted and absorbed energies are written accordingly as:

$$W_{emitted-spt.} = N_n A_{nn'} (h\upsilon),$$

$$W_{emitted-forced} = N_n B_{nn'} (h\upsilon)\rho_\upsilon$$

$$W_{absorbed-forced} = N_{n'} B_{n'n} (h\upsilon)\rho_\upsilon .$$

The energy balance in the closed system of absorbed-emitted-spontaneous radiative phenomena,

$$W_{emitted-spt.} + W_{emitted-forced} = W_{absorbed-forced}$$

leads in first instance with

$$N_n A_{nn'} + N_n B_{nn'} \rho_\upsilon = N_{n'} B_{n'n} \rho_\upsilon$$

from where one yields:

$$\rho_\upsilon = \frac{N_n A_{nn'}}{N_{n'} B_{n'n} - N_n B_{nn'}} = \frac{\dfrac{A_{nn'}}{B_{nn'}}}{\dfrac{N_{n'} B_{n'n}}{N_n B_{nn'}} - 1} = \frac{\dfrac{A_{nn'}}{B_{nn'}}}{\dfrac{B_{n'n}}{B_{nn'}} \exp[\beta(E_n - E_{n'})] - 1}$$

that actually gives:

$$\rho_\upsilon = \frac{\dfrac{A_{nn'}}{B_{nn'}}}{\dfrac{B_{n'n}}{B_{nn'}}\exp[\beta h\upsilon]-1}.$$

Now we have to agree this expression with the Rayleigh-Jens one in the classical limit that is performed in two steps. One is to check out the high-temperature limit through first order expanding the denominator exponential in *h*-Planck constant (i.e. the semi-classical expansion):

$$\infty = \lim_{\beta\to 0}\rho_\upsilon = \frac{\lim\limits_{\beta\to 0}\dfrac{A_{nn'}}{B_{nn'}}}{\lim\limits_{\beta\to 0}\left[\dfrac{B_{n'n}}{B_{nn'}}(1+\beta h\upsilon)-1\right]}$$

from where follows that the forced emission and absorption probabilities are intrinsically equal:

$$B_{nn'} = B_{n'n}$$

since the nominator is non-infinity expression.

In these conditions the spectral density is reloaded in the first order semi-classical *h*-expansion and equated with the classical Rayleigh-Jens expression,

$$\frac{8\pi\upsilon^2}{c^3\beta} = \frac{A_{nn'}}{B_{nn'}}\frac{1}{\beta h\upsilon},$$

that further provides:

$$\frac{A_{nn'}}{B_{nn'}} = \frac{8\pi h\upsilon^3}{c^3}$$

Finally, the spectral density is obtained as in previously Planck approach.