

## Lecture 2: Einstein's Approach of Radiation and the Bohr's Second Postulate

The spontaneous and forced emissions are for the first time solved by Einstein (1917) throughout the introduced coefficients  $A_{nn'}$  and  $B_{nn'}$  (or  $n'n$ ), respectively, relating the probability in unit time once electron going from a state ( $n$ ) of energy  $E_n$  into another one ( $n'$ ) with energy  $E_{n'}$  between which the unit photonic energy exists (is postulated):

$$h\nu = E_n - E_{n'} \quad (1.)$$

As such Einstein accepts at once both the wave quantification as well as the Bohr postulate of transitions between stationary states; while his approach will result in the Planck spectral density  $\rho_\nu$ , the Bohr postulate of quantum transitions follows as being with this occasion demonstrated.

Therefore, while assuming  $N_n$  and  $N_{n'}$  atoms in each of the  $n$  and  $n'$  states in discussion, with  $n > n'$ , within the Maxwell-Boltzmann microstates,

$$N_n = \beta \exp(-\beta E_n),$$

$$N_{n'} = \beta \exp(-\beta E_{n'}),$$

the spontaneous radiant, emitted and absorbed energies are written accordingly as:

$$W_{emitted-spt.} = N_n A_{nn'}(h\nu),$$

$$W_{emitted-forced} = N_n B_{nn'}(h\nu)\rho_\nu$$

$$W_{absorbed-forced} = N_{n'} B_{n'n}(h\nu)\rho_\nu.$$

The energy balance in the closed system of absorbed-emitted-spontaneous radiative phenomena,

$$W_{emitted-spt.} + W_{emitted-forced} = W_{absorbed-forced}$$

leads in first instance with

$$N_n A_{nn'} + N_n B_{nn'}\rho_\nu = N_{n'} B_{n'n}\rho_\nu$$

from where one yields:

$$\rho_\nu = \frac{N_n A_{nn'}}{N_{n'} B_{n'n} - N_n B_{nn'}} = \frac{\frac{A_{nn'}}{B_{nn'}}}{\frac{N_{n'} B_{n'n}}{N_n B_{nn'}} - 1} = \frac{\frac{A_{nn'}}{B_{nn'}}}{\frac{B_{n'n}}{B_{nn'}} \exp[\beta(E_n - E_{n'})] - 1}$$

that actually gives:

$$\rho_\nu = \frac{\frac{A_{m'n'}}{B_{m'n'}}}{\frac{B_{n'n'}}{B_{m'n'}} \exp[\beta h \nu] - 1}.$$

Now we have to agree this expression with the Rayleigh-Jens one in the classical limit that is performed in two steps. One is to check out the high-temperature limit through first order expanding the denominator exponential in  $h$ -Planck constant (i.e. the semi-classical expansion):

$$\infty = \lim_{\beta \rightarrow 0} \rho_\nu = \frac{\lim_{\beta \rightarrow 0} \frac{A_{m'n'}}{B_{m'n'}}}{\lim_{\beta \rightarrow 0} \left[ \frac{B_{n'n'}}{B_{m'n'}} (1 + \beta h \nu) - 1 \right]}$$

from where follows that the forced emission and absorption probabilities are intrinsically equal:

$$B_{m'n'} = B_{n'n'}$$

since the nominator is non-infinity expression.

In these conditions the spectral density is reloaded in the first order semi-classical  $h$ -expansion and equated with the classical Rayleigh-Jens expression,

$$\frac{8\pi\nu^2}{c^3\beta} = \frac{A_{m'n'}}{B_{m'n'}} \frac{1}{\beta h \nu},$$

that further provides:

$$\frac{A_{m'n'}}{B_{m'n'}} = \frac{8\pi h \nu^3}{c^3}$$

Finally, the spectral density is obtained as in previously Planck approach.