

FIZICA MEDIULUI: O Incursiune Spațio-Temporală în Misterele Universului

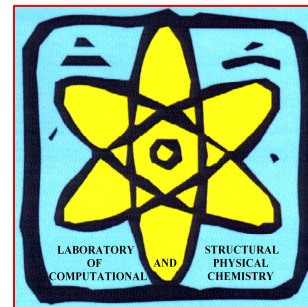
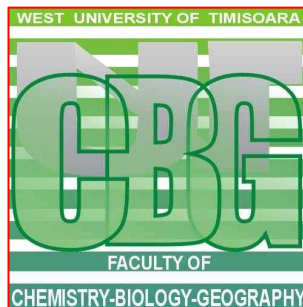
UNIVERSUL STATISTIC: DISTRIBUȚII, PARTICULE, FORȚE

Conf. Dr. Mihai V. PUTZ

*Chemistry Department, West University of Timisoara,
Pestalozzi Street No.16, Timisoara, RO-300115, Romania;
E-mails: mvputz@cbg.uvt.ro or mv_putz@yahoo.com ;
Web: <http://www.mvputz.iqstorm.ro>*

*Member of American Chemical Society
Member of European Society of Mathematical Chemistry*

*Editor in-Chief of Int. J. Chem. Model. (at NOVA Publishers)
Editor in-Chief of Int. J. Environ. Sci. (at SERIALS Publishers)
Guest Editor & Editor of Int. J. Mol. Sci. (at MDPI Organization)*



Sisteme de particule identice

sistem izolat

conservarea numărului de particule $N = \sum_i N_i = ct.$

spațiu mic al fazelor $d\gamma = \underbrace{dp_1 dq_1}_{\approx h} \underbrace{dp_2 dq_2}_{\approx h} \dots \underbrace{dp_f dq_f}_{\approx h} \cong h^f$

ansamblu statistic

micro-stare

macrostrări date de energie $E = \sum_i N_i \varepsilon_i = ct.$

probabilitate termodinamică W $W_E = W_E(N_i, g_i)$

entropie $S = k_B \ln W$

funcția termodinamică de macrostare

$$\Psi = \underbrace{\ln W_E(N_i, g_i)}_{\text{DEZORDINE}} + \underbrace{\alpha \left(N - \sum_i N_i \right) + \beta \left(E - \sum_i N_i \varepsilon_i \right)}_{\text{ORDINE}}$$

FUNCTIE DE MACROSTARE

Echilibrul macrostării

$$\frac{\partial \Psi}{\partial N_i} = 0$$

distribuției

$$N_i = N_i(\varepsilon_i, g_i)$$

Statistica Maxwell-Boltzmann. Funcția de Partiție

$i=1$	$g=2$	
ε	X1 X2	
	<i>sau</i>	
ε		X1 X2
	<i>sau</i>	
ε	X1	X2
	<i>sau</i>	
ε	X2	X1

Stirling $\ln(n!) \cong n \ln n - n$

$$n! \cong e^{n \ln n} e^{-n} = \frac{n^n}{e^n} = \left(\frac{n}{e}\right)^n$$

$$W^B = N! \prod_i \frac{g_i^{N_i}}{N_i!} = N! \prod_i \left(\frac{g_i}{N_i}\right)^{N_i} e^{N_i}$$

$$\Psi^B = \ln W^{MB}(N_i, g_i) + \alpha \left(N - \sum_i N_i\right) + \beta \left(E - \sum_i N_i \varepsilon_i\right)$$

$$= N \ln N - N + \sum_i (N_i \ln g_i - N_i \ln N_i + N_i) + \alpha \left(N - \sum_i N_i\right) + \beta \left(E - \sum_i N_i \varepsilon_i\right)$$

$$0 = \frac{\partial \Psi^B}{\partial N_i} = \ln g_i - \ln N_i - \alpha - \beta \varepsilon_i$$

distribuția Boltzmann

$$N_i^B = g_i \exp(-\alpha - \beta \varepsilon_i)$$

Statistica Maxwell-Boltzmann. Funcția de Partiție

$$N = \sum_i N_i^B = \sum_i g_i \exp(-\alpha - \beta \varepsilon_i) = e^{-\alpha} \sum_i g_i \exp(-\beta \varepsilon_i) = Z e^{-\alpha}$$

funcție de partiție

$$Z = \sum_i g_i \exp(-\beta \varepsilon_i) \quad N_i^B = \frac{N}{Z} \exp(-\beta \varepsilon_i) \quad e^{-\alpha} = \frac{N}{Z}$$

Pentru determinarea parametrului β

$$dE = \sum_i \varepsilon_i dN_i + \sum_i N_i d\varepsilon_i \quad \delta Q = \sum_i \varepsilon_i dN_i$$

$$0 = dN = d\left(\sum_i g_i e^{-\alpha - \beta \varepsilon_i}\right) = \sum_i g_i e^{-\alpha - \beta \varepsilon_i} (-d\alpha - \beta d\varepsilon_i - \varepsilon_i d\beta)$$

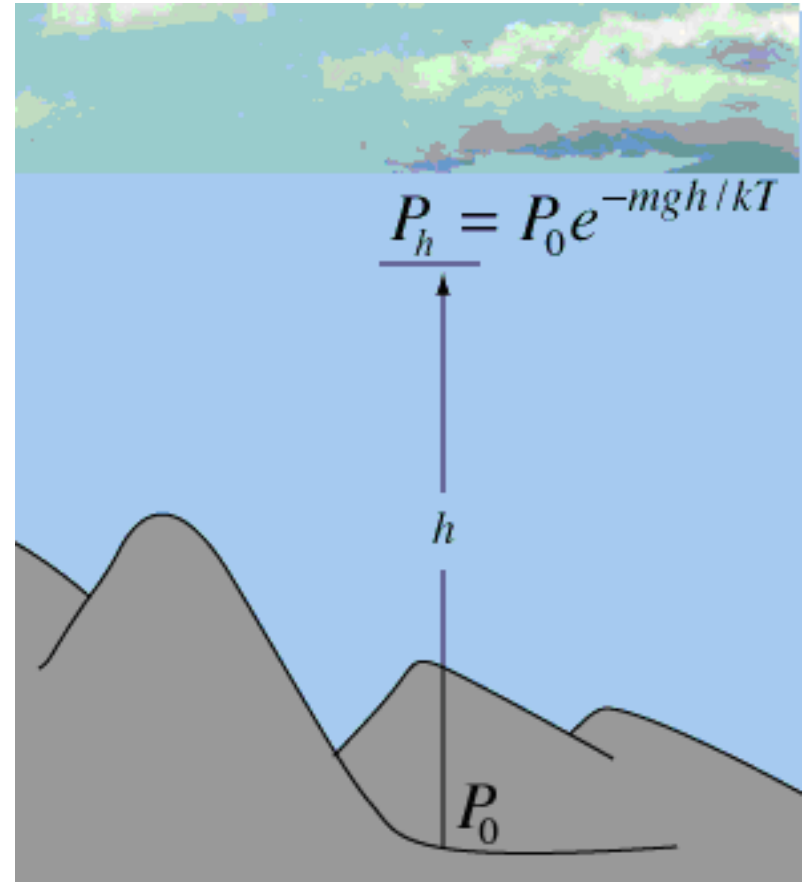
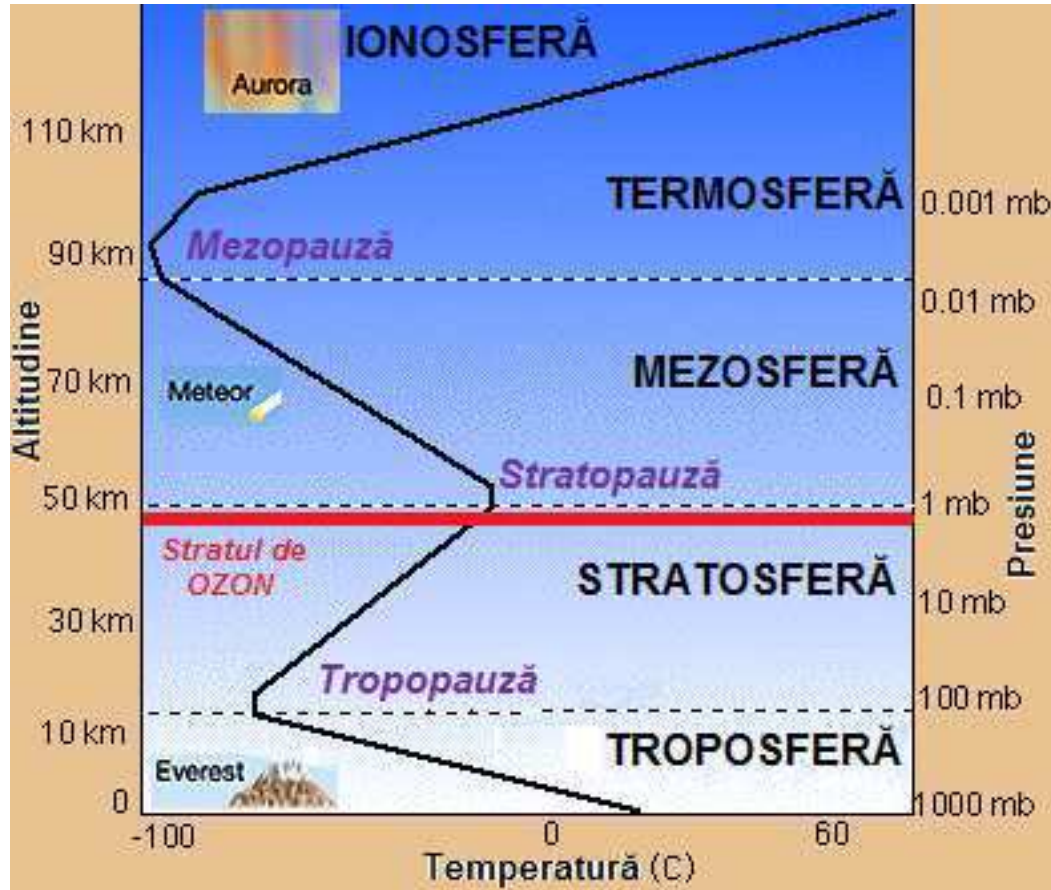
$$\beta \sum_i N_i d\varepsilon_i = -Ed\beta - Nd\alpha$$

$$\begin{aligned} \frac{1}{T} \delta Q &= dS^B = d(k_B \ln W^B) = k_B d \left\{ \ln \left[N! \prod_i e^{N_i(\alpha + \beta \varepsilon_i)} e^{-N_i} \right] \right\} \\ &= k_B d \left[N \ln N - N + \sum_i N_i (\alpha + \beta \varepsilon_i) + \sum_i N_i \right] \\ &= k_B \left(\underbrace{d\alpha \sum_i N_i}_N + \underbrace{\alpha dN}_0 + \underbrace{\beta \sum_i \varepsilon_i dN_i}_{\delta Q} + \underbrace{\beta \sum_i N_i d\varepsilon_i}_{-Ed\beta - Nd\alpha} + \underbrace{d\beta \sum_i N_i \varepsilon_i}_E \right) = k_B \beta \delta Q \end{aligned}$$

$$N_i^B = \frac{N}{Z} \exp\left(-\frac{\varepsilon_i}{k_B T}\right)$$

$$\beta = \frac{1}{k_B T}$$

Formula Barometrică



Formula Barometrică

$$P_z = \frac{F_g}{S} = \frac{(mN_z)g}{S} = \frac{mg}{S} N_z$$

$$\frac{P_{z1}}{P_{z2}} = \frac{N_{z1}}{N_{z2}} \Leftrightarrow P_{z2} N_{z1} = P_{z1} N_{z2}$$

$$P_{z2} \exp\left(-\frac{E_{z1}}{k_B T}\right) = P_{z1} \exp\left(-\frac{E_{z2}}{k_B T}\right)$$

$$N_{z1/z2} = \frac{N}{Z} \exp\left(-\frac{E_{z1/z2}}{k_B T}\right)$$

$$E_{z1/z2} = \frac{1}{2} m v^2 + E_{rot} + E_{vib} + mgz_{1/2}$$

$$P_{z2} \exp\left(-\frac{mgz_1}{k_B T}\right) = P_{z1} \exp\left(-\frac{mgz_2}{k_B T}\right)$$

$$P_h = P_0 \exp\left(-\frac{mgh}{k_B T}\right)$$

$$z_2 - z_1 = h > 0$$

Component	Procent volumic	Presiune parțială	Masă moleculară
	[%]	[mmHg]	[uam]
Azot (N ₂)	78.08	593.4	28.013
Oxigen (O ₂)	20.95	159.2	31.998
Argon (Ar)	0.93	7.1	39.948
CO ₂	0.03	0.2	43.999
	99.99%	759.9 mmHg	28.95 [în medie]

Formula Barometrică

Înălțime [km]	Presiune [torr]	
	Măsurată	Calculată
30	9.5	25
60	0.21	0.8
90	0.0019	0.03

$$T_h = T_0 - bh, \quad T_0 = 288K (15^{\circ}C), \quad b = 6.5K / km$$

$$\begin{aligned}
 P_h &= P_0 \exp\left(-\frac{mgh}{k_B T_0}\right) \\
 &= P_0 \exp\left(-\frac{mg}{k_B T_0} \frac{T_0 - T}{b}\right) = P_0 \left[\exp\left(-\frac{T_0 - T}{T_0}\right) \right]^{\frac{mg}{k_B b}} \\
 &\cong P_0 \left(1 - \frac{bh}{T_0}\right)^{\frac{mg}{k_B b}}
 \end{aligned}$$

Statistica Fermi-Dirac

<i>Nivelul i</i>	g_i					
ε_i	X1		X2		X3	X4

$$W^{FD} = \prod_i \frac{g_i!}{N_i!(g_i - N_i)!} = \prod_i \frac{g_i^{g_i}}{N_i^{N_i}(g_i - N_i)^{g_i - N_i}} = \prod_i \frac{\left(\frac{g_i}{N_i}\right)^{g_i}}{\left(\frac{g_i}{N_i} - 1\right)^{g_i - N_i}}$$

$$\Psi^{FD} = \sum_i [g_i \ln g_i - N_i \ln N_i - (g_i - N_i) \ln (g_i - N_i)] + \alpha \left(N - \sum_i N_i \right) + \beta \left(E - \sum_i N_i \varepsilon_i \right)$$

$$0 = \frac{\partial \Psi^{FD}}{\partial N_i} = \ln N_i - \ln (g_i - N_i) - \alpha - \beta \varepsilon_i$$

$$N_i^{FD} = \frac{g_i}{\exp(\alpha + \beta \varepsilon_i) + 1}$$

Statistica Fermi-Dirac

$$dS = \frac{1}{T} dQ = \frac{1}{T} (dE + pdV - \mu dN)$$

$$= \left(\frac{\partial S}{\partial E} \right)_{V,N} dE + \left(\frac{\partial S}{\partial V} \right)_{E,N} dV + \left(\frac{\partial S}{\partial N} \right)_{E,V} dN$$

$$\boxed{\frac{\partial S}{\partial E} = \frac{1}{T}}$$

$$\boxed{\frac{\partial S}{\partial N} = -\frac{\mu}{T}}$$

$$S^{FD} = k_B \ln W^{FD} = k_B \ln \left[\prod_i \frac{(e^{\alpha + \beta \varepsilon_i} + 1)^{g_i}}{(e^{\alpha + \beta \varepsilon_i})^{g_i - N_i}} \right] = k_B \ln \left[\prod_i (e^{\alpha + \beta \varepsilon_i})^{N_i} (1 + e^{-\alpha - \beta \varepsilon_i})^{g_i} \right]$$

$$= k_B \left\{ \sum_i N_i (\alpha + \beta \varepsilon_i) + \sum_i g_i \ln(1 + e^{-\alpha - \beta \varepsilon_i}) \right\} = k_B \left[\alpha N + \beta E + \sum_i g_i \ln(1 + e^{-\alpha - \beta \varepsilon_i}) \right]$$

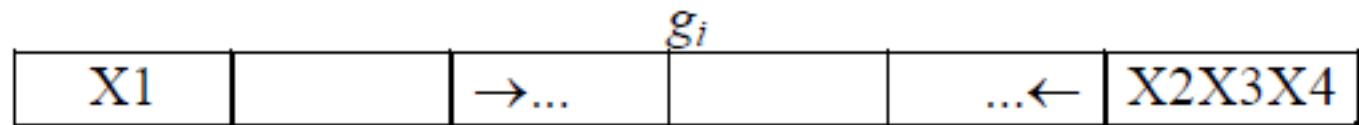
$$\alpha = -\frac{\mu}{k_B T} \quad \beta = \frac{1}{k_B T}$$

$$\boxed{N_i^{FD} = \frac{g_i}{\exp\left(\frac{\varepsilon_i - \mu}{k_B T}\right) + 1}}$$

Statistica Bose-Einstein

Nivelul i

ε_i



$$W^{BE} = \prod_i \frac{(N_i + g_i - 1)!}{N_i! (g_i - 1)!} \cong \prod_i \frac{(N_i + g_i)!}{N_i! g_i!} \cong \prod_i \frac{(N_i + g_i)^{N_i + g_i}}{N_i^{N_i} g_i^{g_i}} = \prod_i \left(\frac{N_i}{g_i} \right)^{g_i} \left(1 + \frac{g_i}{N_i} \right)^{N_i + g_i}$$

$$\Psi^{BE} = \sum_i [(N_i + g_i) \ln(N_i + g_i) - N_i \ln N_i - g_i \ln g_i] + \alpha \left(N - \sum_i N_i \right) + \beta \left(E - \sum_i N_i \varepsilon_i \right)$$

$$0 = \frac{\partial \Psi^{BE}}{\partial N_i} = \ln(N_i + g_i) - \ln N_i - \alpha - \beta \varepsilon_i$$

$$N_i^{BE} = \frac{g_i}{\exp(\alpha + \beta \varepsilon_i) - 1}$$

$$S^{BE} = k_B \ln W^{BE} = k_B \left[\alpha N + \beta E - \sum_i g_i \ln(1 - e^{-\alpha - \beta \varepsilon_i}) \right]$$

$$-\frac{\mu}{T} = \frac{\partial S^{BE}}{\partial N} = \alpha k_B \Rightarrow \alpha = -\frac{\mu}{k_B T}$$

$$\frac{1}{T} = \frac{\partial S^{BE}}{\partial E} = k_B \beta \Rightarrow \beta = \frac{1}{k_B T}$$

$$N_i^{BE} = \frac{g_i}{\exp\left(\frac{\varepsilon_i - \mu}{k_B T}\right) - 1}$$

TOE (Theory of Everything)

$$\varepsilon_i \cong nk_B T, n \gg 1 \quad \lim_{T \rightarrow \infty} N_i^{FD/BE} = N_i^B$$

FERMIONI ELEMENTARI

A. Fermionii ușori: leptonii

$$Q = -1 : \begin{pmatrix} e^- \\ \nu_e \end{pmatrix}, \begin{pmatrix} \mu^- \\ \nu_\mu \end{pmatrix}, \begin{pmatrix} \tau^- \\ \nu_\tau \end{pmatrix}$$

B. Fermionii grei: quarkurile

$$Q = +2/3 : \begin{pmatrix} u \\ d \end{pmatrix}, \begin{pmatrix} c \\ s \end{pmatrix}, \begin{pmatrix} t \\ b \end{pmatrix}$$

up
charm
top / truth
down
strange
bottom / beauty

FORȚE FUNDAMENTALE

A. Interacția Tare

GLUON

- Cu spinul $s(g)=1$
- Cu masă zero $m(g)=0$

B. Interacția Electro-Magnetică

FOTON- γ

- Cu spinul $s(\gamma)=1$
- Cu masă zero $m_0(\gamma)=0$

C. Interacția Slabă

WEAKONI

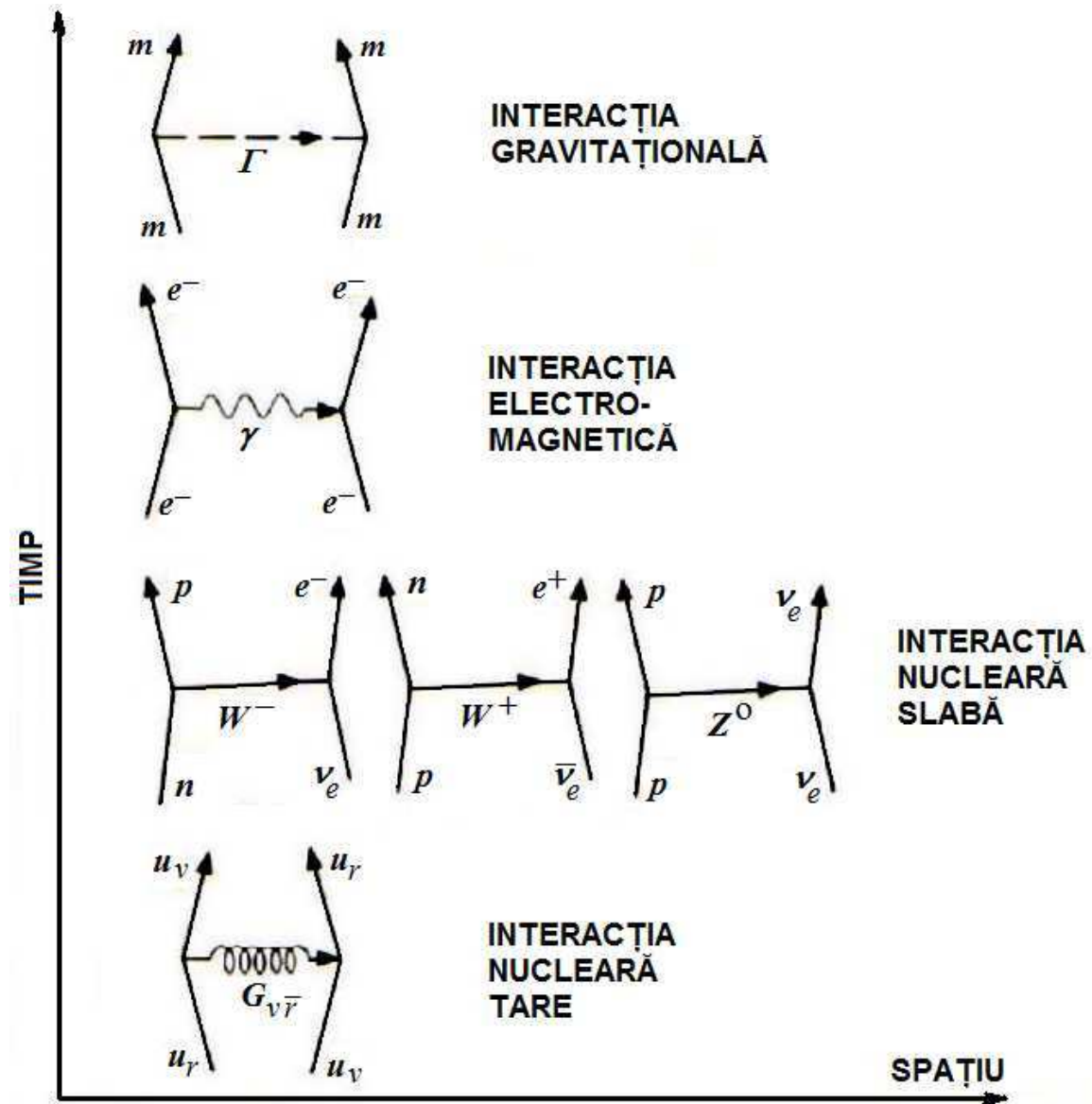
$W^\pm \quad Z^0$

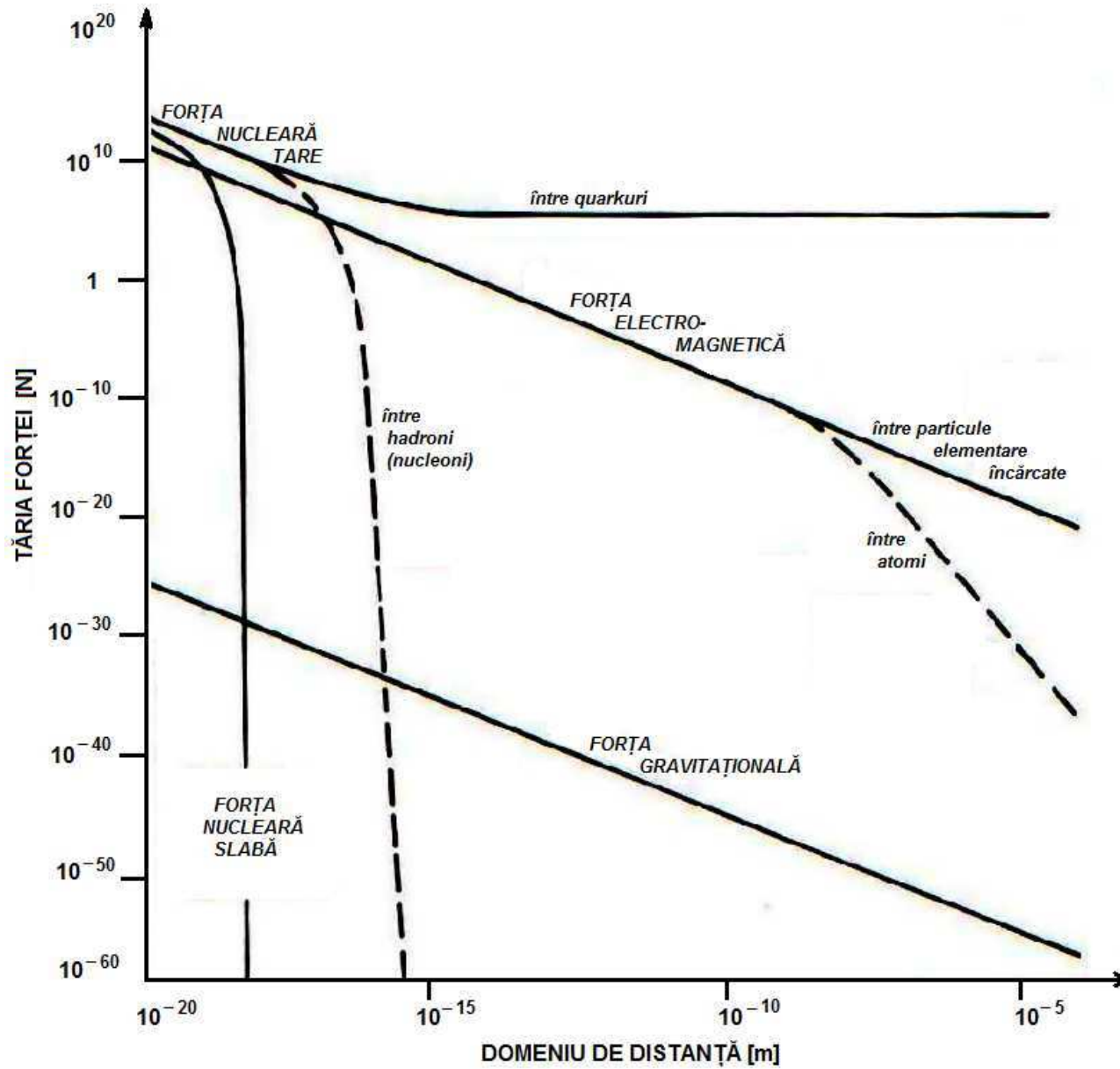
- Cu spinul $s(W^\pm, Z^0)=1$
- Cu masele $m_0(W^\pm) \sim 84 \text{ GeV}$
 $m_0(Z^0) \sim 94 \text{ GeV}$

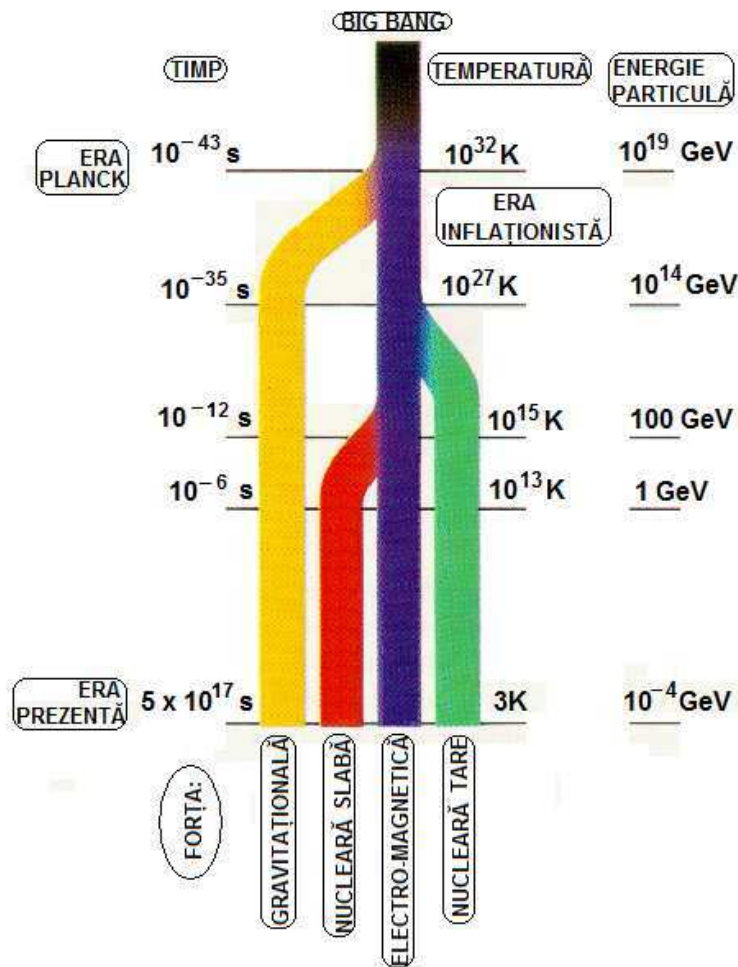
D. Interacția Gravitațională

GRAVITON- Γ

- Cu spinul $s(\Gamma)=2$
- Cu masă de repaos zero $m_0(\Gamma)=0$







unificarea macro-micro cosmosului

$$ma = G \frac{mM}{r^2}$$

$$G = \frac{ar^2}{M}$$

$$G = \frac{L_p c^2}{M_p}$$

$$p\lambda = h$$

$$(M_p c) L_p = h$$

$$M_p = \sqrt{\frac{ch}{G}} \cong 5 \cdot 10^{-8} [kg]$$

$$L_p = \sqrt{\frac{Gh}{c^3}} \cong 4 \cdot 10^{-35} [m]$$

$$t_p = \frac{L_p}{c} = \sqrt{\frac{Gh}{c^5}} \cong 10^{-43} [s]$$

$$E_p = \frac{h}{t_p} = \sqrt{\frac{hc^5}{G}} \cong 5 \cdot 10^9 [J] \cong 10^{19} [GeV]$$

$$T_p = \frac{E_p}{k_B} = \sqrt{\frac{hc^5}{Gk_B^2}} \cong 3 \cdot 10^{32} [K]$$

$$\rho_p = \frac{M_p}{L_p^3} = \frac{c^5}{hG^2} \cong 10^{96} [kg / m^3]$$



Paul Adrien Maurice Dirac



Satyendra Nath Bose