FIZICA MEDIULUI:
O Incursiune Spațio-Temporală în Misterele Universului

UNIVERSUL STATISTIC: DISTRIBUȚII, PARTICULE, FORȚE

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Sisteme de particule identice

sistem izolat

conservarea numărului de particule $N = \sum_i N_i = ct.$

spațiu mic al fazelor $d\gamma = \underbrace{dp_1 dq_1}_{\approx h} \underbrace{dp_2 dq_2}_{\approx h} \ldots \underbrace{dp_f dq_f}_{\approx h} \cong h^f$

ansamblu statistic

micro-stare

macrostrări date de energie $E = \sum_i N_i \varepsilon_i = ct.$

probabilitate termodinamică W $W_E = W_E(N_i, g_i)$

entropie $S = k_B \ln W$

funcția termodinamică de macrostare

$$\Psi = \underbrace{\underbrace{\ln W_E(N_i, g_i)}_{DEZORDINE} + \underbrace{\alpha\left(N - \sum_i N_i\right) + \beta\left(E - \sum_i N_i \varepsilon_i\right)}_{ORDINE}}_{FUNCTIE\ DE\ MACROSTARE}$$

Echilibrul macrostării $\dfrac{\partial \Psi}{\partial N_i} = 0$

distribuției

$N_i = N_i(\varepsilon_i, g_i)$

Statistica Maxwell-Boltzmann. Funcţia de Partiţie

i=1 g=2

ε	X1 X2	

sau

ε		X1 X2

sau

ε	X1	X2

sau

ε	X2	X1

Stirling

$$\ln(n!) \cong n \ln n - n$$

$$n! \cong e^{n \ln n} e^{-n} = \frac{n^n}{e^n} = \left(\frac{n}{e}\right)^n$$

$$W^B = N! \prod_i \frac{g_i^{N_i}}{N_i!} = N! \prod_i \left(\frac{g_i}{N_i}\right)^{N_i} e^{N_i}$$

$$\Psi^B = \ln W^{MB}(N_i, g_i) + \alpha\left(N - \sum_i N_i\right) + \beta\left(E - \sum_i N_i \varepsilon_i\right)$$

$$= N \ln N - N + \sum_i (N_i \ln g_i - N_i \ln N_i + N_i) + \alpha\left(N - \sum_i N_i\right) + \beta\left(E - \sum_i N_i \varepsilon_i\right)$$

$$0 = \frac{\Psi^B}{\partial N_i} = \ln g_i - \ln N_i - \alpha - \beta\varepsilon_i$$

distribuţia Boltzmann

$$N_i^B = g_i \exp(-\alpha - \beta\varepsilon_i)$$

Statistica Maxwell-Boltzmann. Funcția de Partiție

$$N=\sum_i N_i^B=\sum_i g_i\exp(-\alpha-\beta\varepsilon_i)=e^{-\alpha}\sum_i g_i\exp(-\beta\varepsilon_i)=Ze^{-\alpha}$$

funcție de partiție

$$Z=\sum_i g_i\exp(-\beta\varepsilon_i) \qquad N_i^B=\frac{N}{Z}\exp(-\beta\varepsilon_i) \qquad e^{-\alpha}=\frac{N}{Z}$$

Pentru determinarea parametrului β

$$dE=\sum_i \varepsilon_i dN_i+\sum_i N_i d\varepsilon_i \qquad đQ=\sum_i \varepsilon_i dN_i$$

$$0=dN=d\left(\sum_i g_i e^{-\alpha-\beta\varepsilon_i}\right)=\sum_i g_i e^{-\alpha-\beta\varepsilon_i}\left(-d\alpha-\beta d\varepsilon_i-\varepsilon_i d\beta\right)$$

$$\beta\sum_i N_i d\varepsilon_i=-Ed\beta-Nd\alpha$$

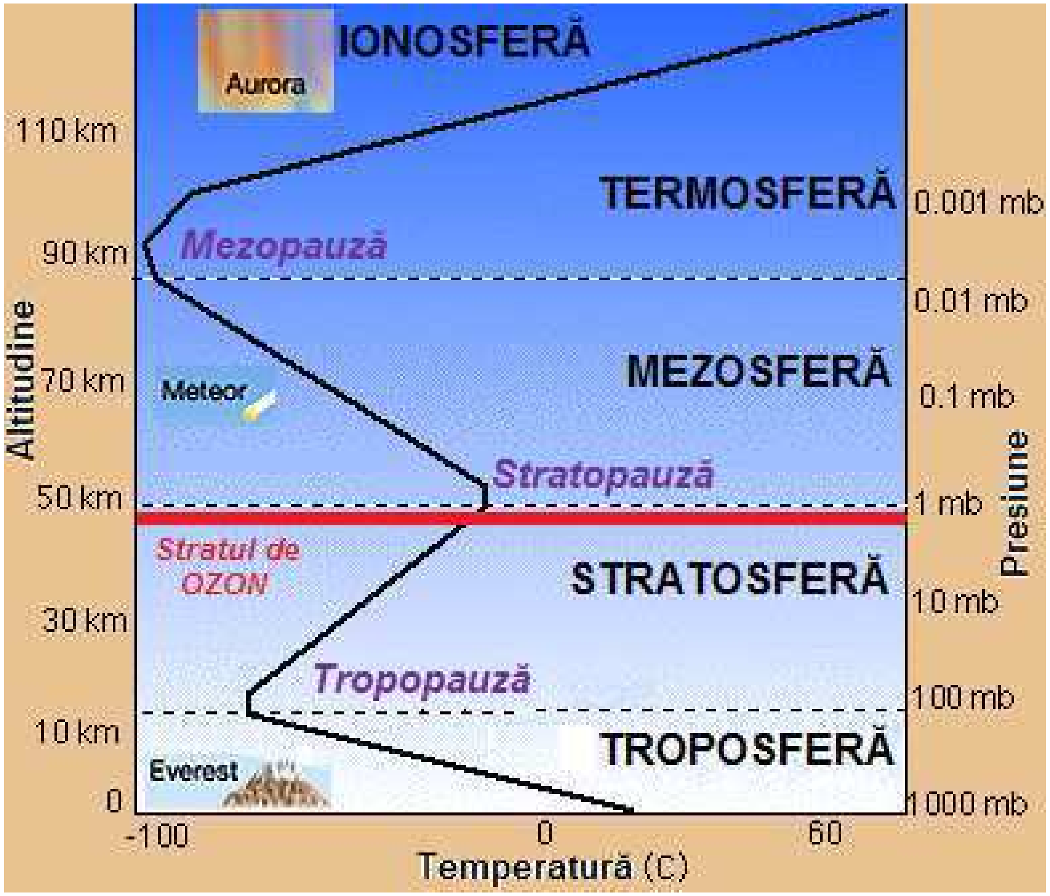
$$\frac{1}{T}đQ=dS^B=d\left(k_B\ln W^B\right)=k_B d\left\{\ln\left[N!\prod_i e^{N_i(\alpha+\beta\varepsilon_i)}e^{N_i}\right]\right\}$$

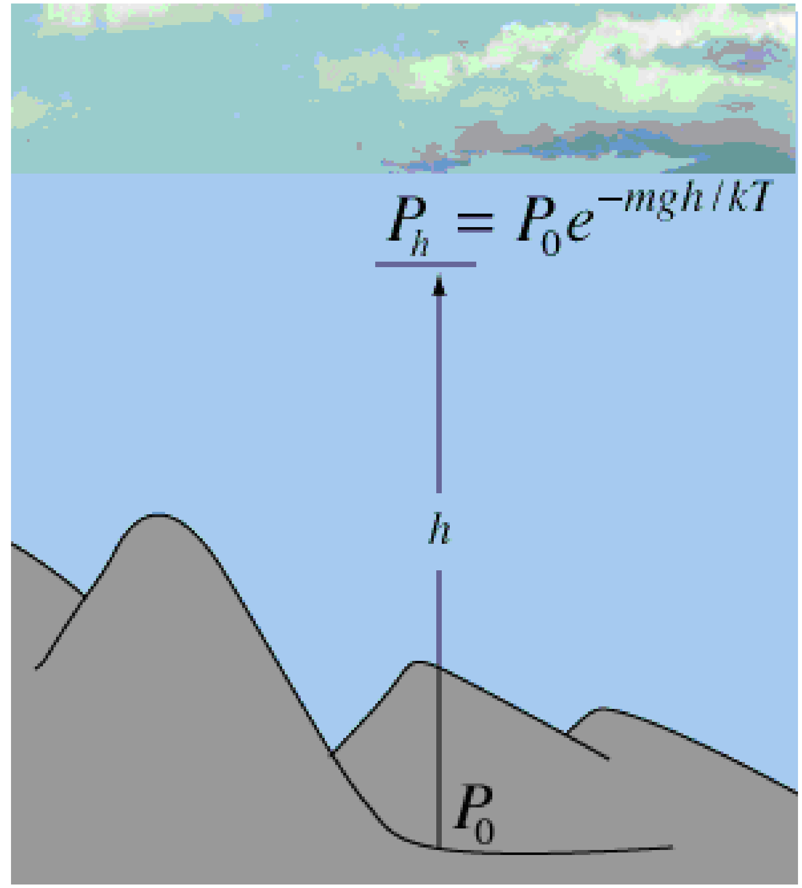
$$=k_B d\left[N\ln N-N+\sum_i N_i(\alpha+\beta\varepsilon_i)+\sum_i N_i\right]$$

$$=k_B\Big(d\alpha\underbrace{\sum_i N_i}_{N}+\alpha\underbrace{dN}_{0}+\beta\underbrace{\sum_i \varepsilon_i dN_i}_{đQ}+\beta\underbrace{\sum_i N_i d\varepsilon_i}_{-Ed\beta-Nd\alpha}+d\beta\underbrace{\sum_i N_i\varepsilon_i}_{E}\Big)=k_B\beta đQ$$

$$\boxed{N_i^B=\frac{N}{Z}\exp\left(-\frac{\varepsilon_i}{k_BT}\right)} \qquad \beta=\frac{1}{k_BT}$$

Formula Barometrică





Formula Barometrică

$$P_z = \frac{F_g}{S} = \frac{(mN_z)g}{S} = \frac{mg}{S}N_z$$

$$\frac{P_{z1}}{P_{z2}} = \frac{N_{z1}}{N_{z2}} \Leftrightarrow P_{z2}N_{z1} = P_{z1}N_{z2}$$

$$P_{z2}\exp\left(-\frac{E_{z1}}{k_B T}\right) = P_{z1}\exp\left(-\frac{E_{z2}}{k_B T}\right)$$

$$N_{z1/z2} = \frac{N}{Z}\exp\left(-\frac{E_{z1/z2}}{k_B T}\right)$$

$$E_{z1/z2} = \frac{1}{2}mv^2 + E_{rot} + E_{vib} + mgz_{1/2}$$

$$P_{z2}\exp\left(-\frac{mgz_1}{k_B T}\right) = P_{z1}\exp\left(-\frac{mgz_2}{k_B T}\right)$$

$$P_h = P_0\exp\left(-\frac{mgh}{k_B T}\right)$$

$$z_2 - z_1 = h > 0$$

Component	Procent volumic [%]	Presiune parțială [mmHg]	Masă moleculară [uam]
Azot (N_2)	78.08	593.4	28.013
Oxigen (O_2)	20.95	159.2	31.998
Argon (Ar)	0.93	7.1	39.948
CO_2	0.03	0.2	43.999
	99.99%	759.9 mmHg	28.95 [în medie]

Formula Barometrică

Înălțime [km]	Presiune [torr]	
	Măsurată	Calculată
30	9.5	25
60	0.21	0.8
90	0.0019	0.03

$$T_h = T_0 - bh\,,\ T_0 = 288K(15^0C),\ b = 6.5K/km$$

$$P_h = P_0 \exp\left(-\frac{mgh}{k_B T_0}\right)$$

$$= P_0 \exp\left(-\frac{mg}{k_B T_0}\frac{T_0 - T}{b}\right) = P_0\left[\exp\left(-\frac{T_0 - T}{T_0}\right)\right]^{\frac{mg}{k_B b}}$$

$$\cong P_0\left(1 - \frac{bh}{T_0}\right)^{\frac{mg}{k_B b}}$$

Statistica Fermi-Dirac

Nivelul i	g_i					
ε_i	X1		X2		X3	X4

$$W^{FD} = \prod_i \frac{g_i!}{N_i!(g_i - N_i)!} = \prod_i \frac{g_i^{g_i}}{N_i^{N_i}(g_i - N_i)^{g_i - N_i}} = \prod_i \frac{\left(\frac{g_i}{N_i}\right)^{g_i}}{\left(\frac{g_i}{N_i} - 1\right)^{g_i - N_i}}$$

$$\Psi^{FD} = \sum_i \left[g_i \ln g_i - N_i \ln N_i - (g_i - N_i)\ln(g_i - N_i)\right] + \alpha\left(N - \sum_i N_i\right) + \beta\left(E - \sum_i N_i \varepsilon_i\right)$$

$$0 = \frac{\Psi^{FB}}{\partial N_i} = \ln N_i - \ln(g_i - N_i) - \alpha - \beta\varepsilon_i$$

$$N_i^{FD} = \frac{g_i}{\exp(\alpha + \beta\varepsilon_i) + 1}$$

Statistica Fermi-Dirac

$$dS = \frac{1}{T} đQ \quad = \frac{1}{T}\left(dE + pdV - \mu dN\right)$$

$$= \left(\frac{\partial S}{\partial E}\right)_{V,N} dE + \left(\frac{\partial S}{\partial V}\right)_{E,N} dV + \left(\frac{\partial S}{\partial N}\right)_{E,V} dN$$

$$\boxed{\frac{\partial S}{\partial E} = \frac{1}{T}} \qquad \boxed{\frac{\partial S}{\partial N} = -\frac{\mu}{T}}$$

$$S^{FD} = k_B \ln W^{FD} = k_B \ln\left[\prod_i \frac{\left(e^{\alpha+\beta\varepsilon_i}+1\right)^{g_i}}{\left(e^{\alpha+\beta\varepsilon_i}\right)^{g_i-N_i}}\right] = k_B \ln\left[\prod_i \left(e^{\alpha+\beta\varepsilon_i}\right)^{N_i}\left(1+e^{-\alpha-\beta\varepsilon_i}\right)^{g_i}\right]$$

$$= k_B\left\{\sum_i N_i(\alpha+\beta\varepsilon_i) + \sum_i g_i \ln\left(1+e^{-\alpha-\beta\varepsilon_i}\right)\right\} = k_B\left[\alpha N + \beta E + \sum_i g_i \ln\left(1+e^{-\alpha-\beta\varepsilon_i}\right)\right]$$

$$\alpha = -\frac{\mu}{k_B T} \qquad \beta = \frac{1}{k_B T} \qquad \boxed{N_i^{FD} = \frac{g_i}{\exp\left(\frac{\varepsilon_i - \mu}{k_B T}\right)+1}}$$

Statistica Bose-Einstein

Nivelul i $\qquad\qquad\qquad\qquad g_i$

ε_i

X1		→...		...←	X2X3X4

$$W^{BE} = \prod_i \frac{(N_i + g_i - 1)!}{N_i!(g_i - 1)!} \cong \prod_i \frac{(N_i + g_i)!}{N_i! g_i!} \cong \prod_i \frac{(N_i + g_i)^{N_i + g_i}}{N_i^{N_i} g_i^{g_i}} = \prod_i \left(\frac{N_i}{g_i}\right)^{g_i} \left(1 + \frac{g_i}{N_i}\right)^{N_i + g_i}$$

$$\Psi^{BE} = \sum_i \left[(N_i + g_i)\ln(N_i + g_i) - N_i \ln N_i - g_i \ln g_i\right] + \alpha\left(N - \sum_i N_i\right) + \beta\left(E - \sum_i N_i \varepsilon_i\right)$$

$$0 = \frac{\Psi^{BE}}{\partial N_i} = \ln(N_i + g_i) - \ln N_i - \alpha - \beta\varepsilon_i \qquad\qquad N_i^{BE} = \frac{g_i}{\exp(\alpha + \beta\varepsilon_i) - 1}$$

$$S^{BE} = k_B \ln W^{FD} = k_B\left[\alpha N + \beta E - \sum_i g_i \ln\left(1 - e^{-\alpha - \beta\varepsilon_i}\right)\right]$$

$$-\frac{\mu}{T} = \frac{\partial S^{BE}}{\partial N} = \alpha k_B \Rightarrow \alpha = -\frac{\mu}{k_B T}$$

$$\frac{1}{T} = \frac{\partial S^{BE}}{\partial E} = k_B \beta \Rightarrow \beta = \frac{1}{k_B T}$$

$$\boxed{N_i^{BE} = \frac{g_i}{\exp\left(\frac{\varepsilon_i - \mu}{k_B T}\right) - 1}}$$

TOE (Theory of Everything)

$$\varepsilon_i \cong nk_B T \, , \, n>>1 \qquad \lim_{T\to\infty} N_i^{FD/BE} = N_i^B$$

FERMIONI ELEMENTARI

A. Fermionii ușori: leptonii

$$\begin{matrix} Q=-1 \\ Q=0 \end{matrix} : \begin{pmatrix} e^- \\ \upsilon_e \end{pmatrix}, \begin{pmatrix} \mu^- \\ \upsilon_\mu \end{pmatrix}, \begin{pmatrix} \tau^- \\ \upsilon_\tau \end{pmatrix}$$

B. Fermionii grei: quarkurile

$$\begin{matrix} Q=+2/3 \\ Q=-1/3 \end{matrix} : \underbrace{\begin{pmatrix} u \\ d \end{pmatrix}}_{\begin{pmatrix} up \\ down \end{pmatrix}}, \underbrace{\begin{pmatrix} c \\ s \end{pmatrix}}_{\begin{pmatrix} charm \\ strange \end{pmatrix}}, \underbrace{\begin{pmatrix} t \\ b \end{pmatrix}}_{\begin{pmatrix} top/truth \\ botton/beauty \end{pmatrix}}$$

FORȚE FUNDAMENTALE

A. Interacția Tare

GLUON
- Cu spinul s(g)=1
- Cu masă zero m(g)=0

B. Interacția Electro-Magnetică

FOTON-γ
- Cu spinul $s(\gamma)=1$
- Cu masă zero $m_0(\gamma)=0$

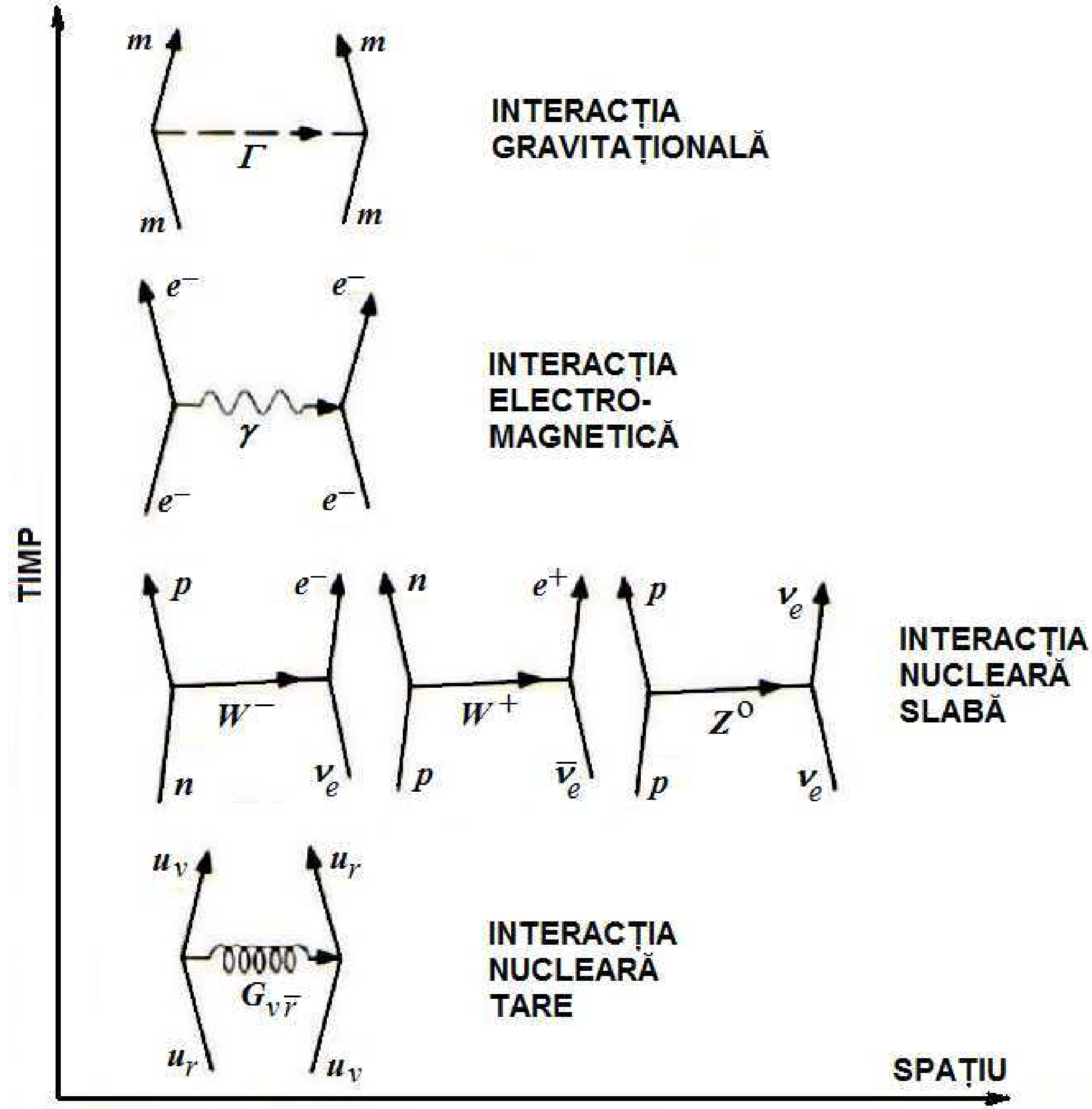
C. Interacția Slabă

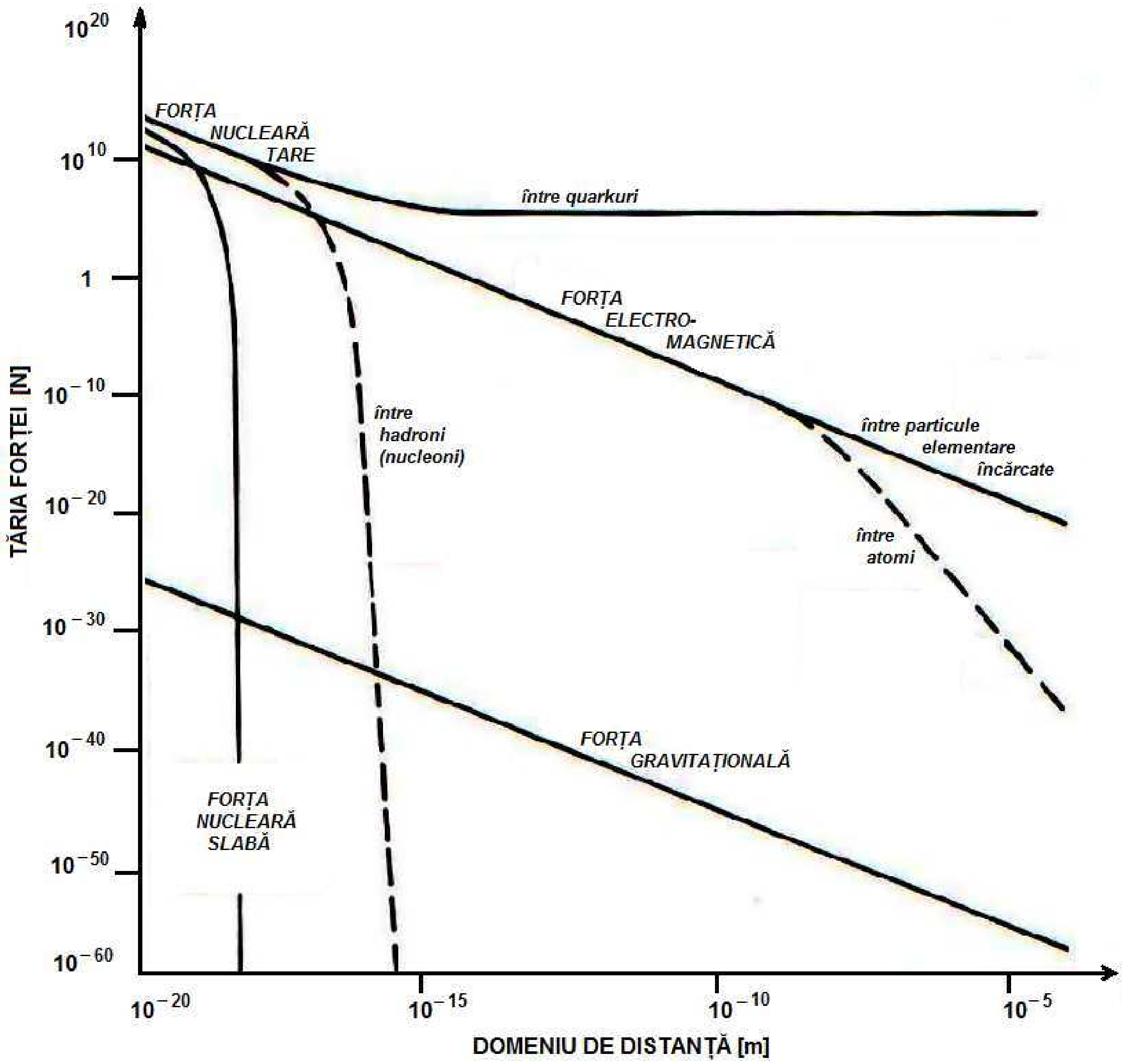
WEAKONI

$W^{\pm} \quad Z^0$
- Cu spinul $s(W^{\pm}, Z^0)=1$
- Cu masele $m_0(W^{\pm})\sim 84$ GeV
  $m_0(Z^0)\sim 94$ GeV

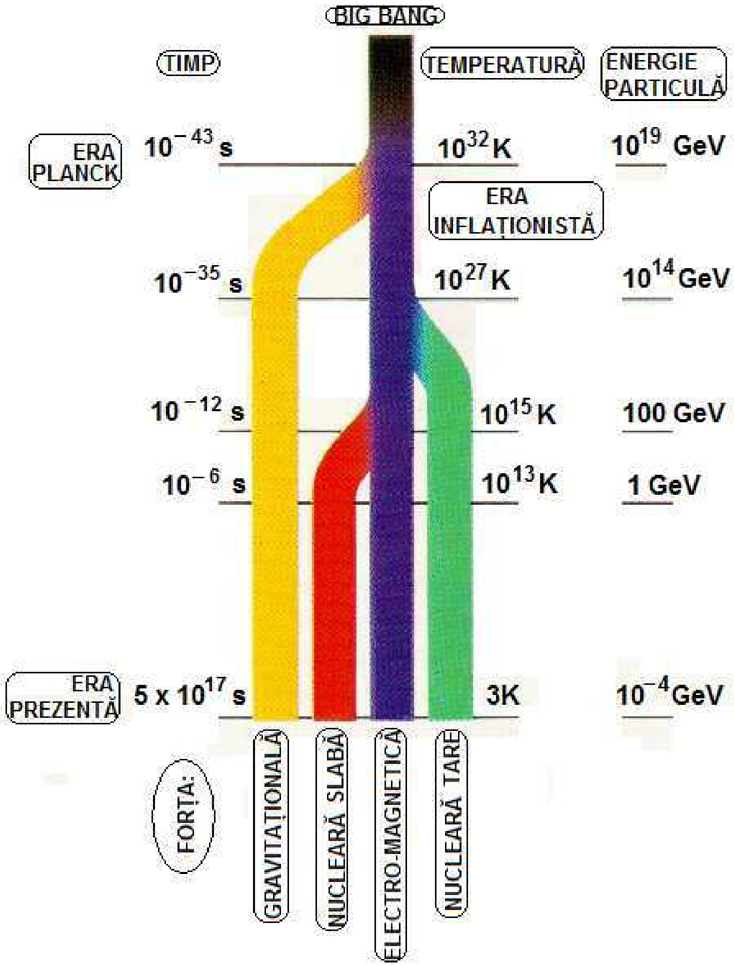
D. Interacția Gravitațională

GRAVITON-Γ
- Cu spinul $s(\Gamma)=2$
- Cu masă de repaos zero $m_0(\Gamma)=0$


m
m
Γ
m
m
INTERACȚIA
GRAVITAȚIONALĂ
e⁻
e⁻
γ
e⁻
e⁻
INTERACȚIA
ELECTRO-
MAGNETICĂ
TIMP
p
e⁻
W⁻
n
νe
n
e⁺
W⁺
p
ν̄e
p
νe
Z⁰
p
νe
INTERACȚIA
NUCLEARĂ
SLABĂ
uv
ur
Gvr̄
ur
uv
INTERACȚIA
NUCLEARĂ
TARE
SPAȚIU



10^{20}
10^{10}
1
10^{-10}
10^{-20}
10^{-30}
10^{-40}
10^{-50}
10^{-60}
TĂRIA FORȚEI [N]
FORȚA NUCLEARĂ TARE
între quarkuri
între hadroni (nucleoni)
FORȚA ELECTRO-MAGNETICĂ
între particule elementare încărcate
între atomi
FORȚA GRAVITAȚIONALĂ
FORȚA NUCLEARĂ SLABĂ
10^{-20}
10^{-15}
10^{-10}
10^{-5}
DOMENIU DE DISTANȚĂ [m]



$$ma = G\frac{mM}{r^2} \qquad G = \frac{ar^2}{M} \qquad G = \frac{L_P c^2}{M_P}$$

$$p\lambda = h \qquad (M_P c)L_P = h$$

$$M_P = \sqrt{\frac{ch}{G}} \cong 5\cdot 10^{-8}[kg]$$

$$L_P = \sqrt{\frac{Gh}{c^3}} \cong 4\cdot 10^{-35}[m]$$

$$t_P = \frac{L_P}{c} = \sqrt{\frac{Gh}{c^5}} \cong 10^{-43}[s]$$

$$E_P = \frac{h}{t_P} = \sqrt{\frac{hc^5}{G}} \cong 5\cdot 10^{9}[J] \cong 10^{19}[GeV]$$

$$T_P = \frac{E_P}{k_B} = \sqrt{\frac{hc^5}{Gk_b^2}} \cong 3\cdot 10^{32}[K]$$

$$\rho_P = \frac{M_P}{L_P^3} = \frac{c^5}{hG^2} \cong 10^{96}[kg/m^3]$$

unificarea macro-micro cosmosului

Paul AdrienMaurice Dirac

Satyendra Nath Bose