FIZICA MEDIULUI:
O Incursiune Spațio-Temporală în Misterele Universului

EVOLUȚIA STELARĂ. PITICELE ALBE

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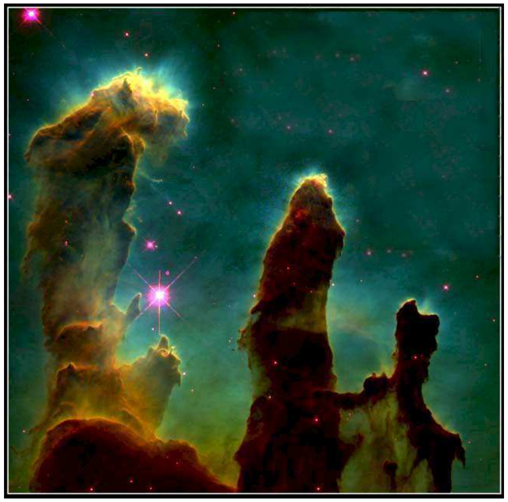
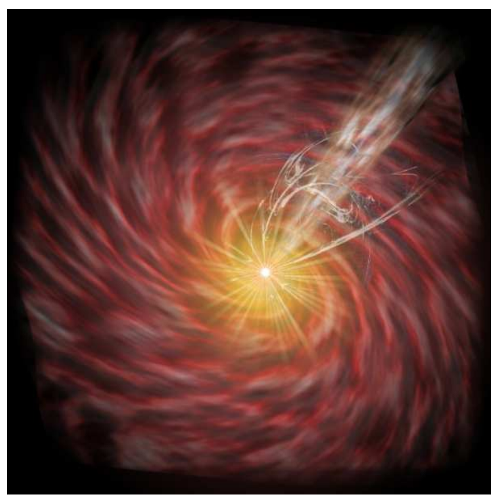
Ciclul stelar

I. Nebuloasa pre- și inter- stelară

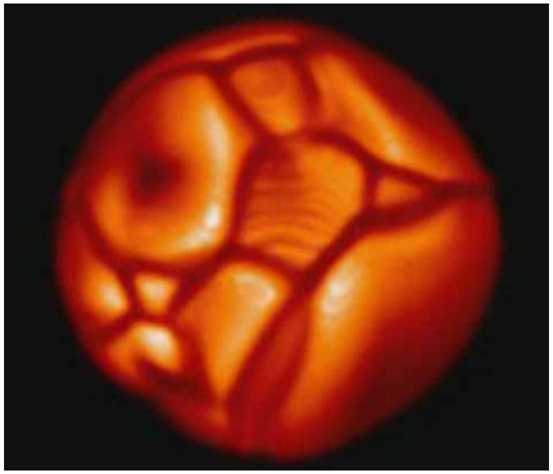


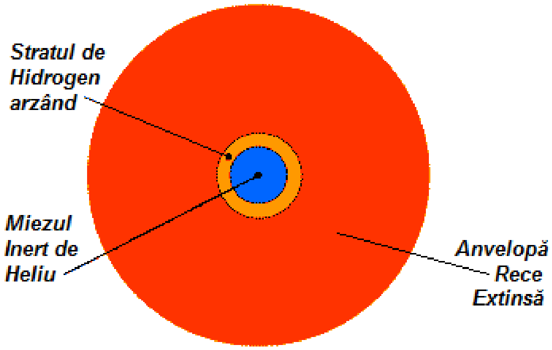
II. Stadiul proto-stelar.

III. Stadiul stelelor din secvența principală (de tip Solar)

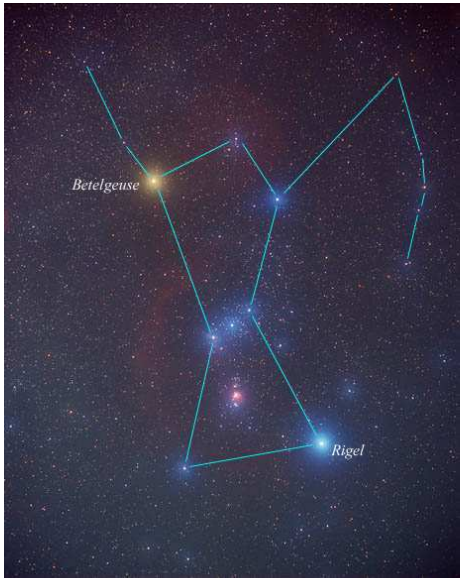


IV. Stadiul stelelor gigantice și supergigantice roșii.

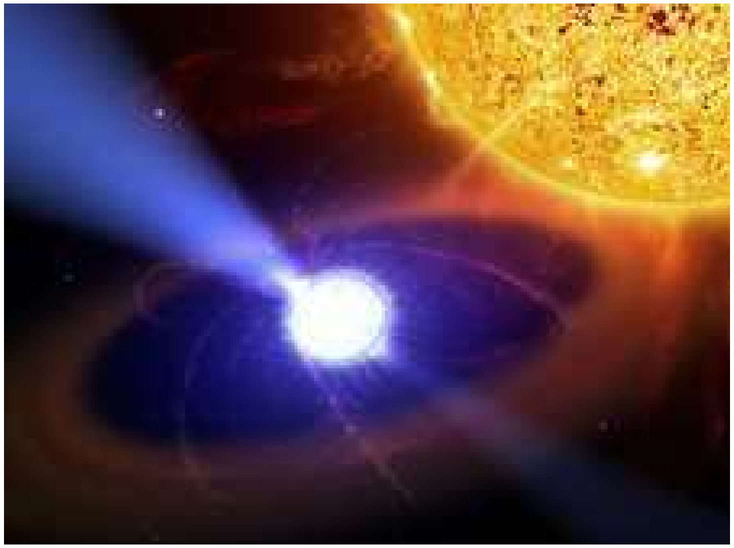


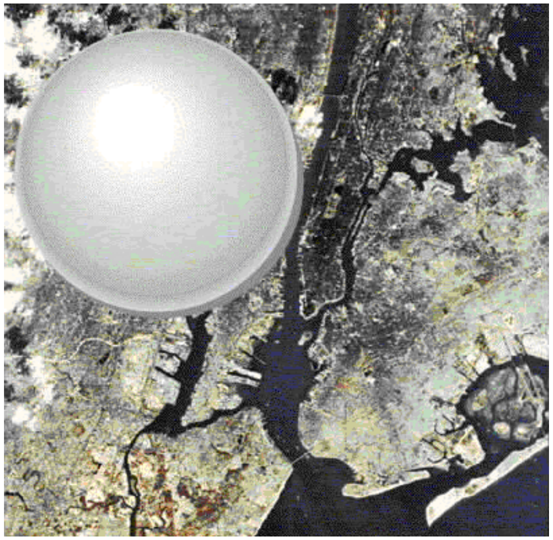


V. Stadiul nebulei planetare



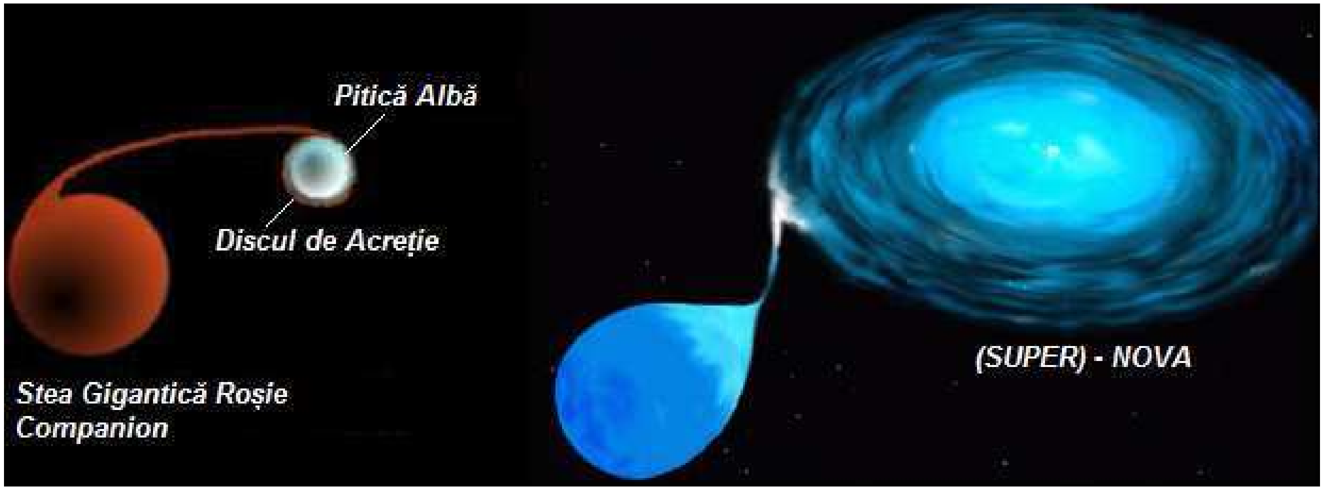
VI. Stadiul stelelor pitice albe



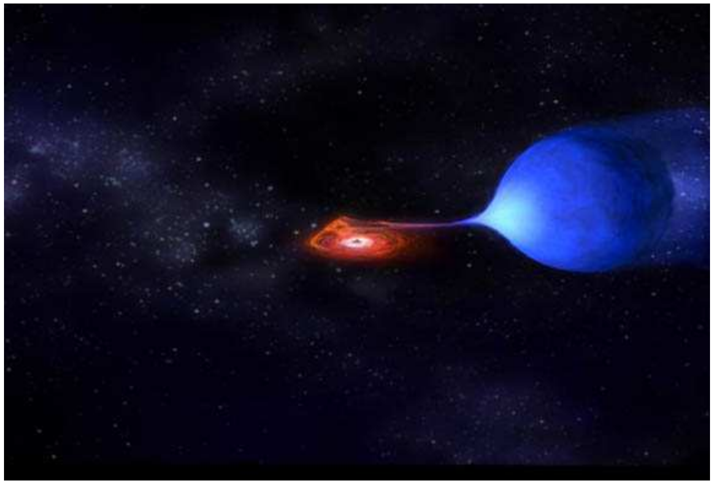


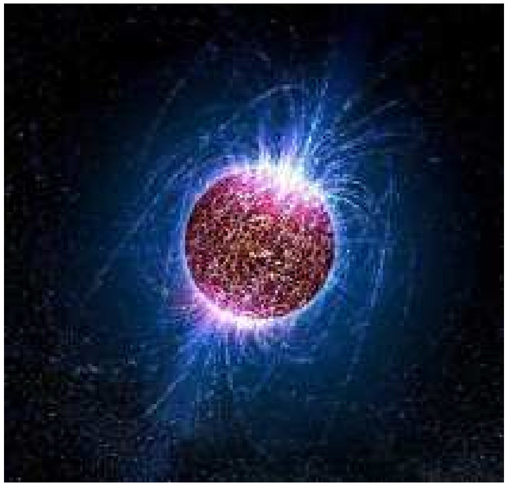
VII. Stadiul stelelor pitice negre

VIII. Stadiul NOVA & SUPER-NOVA

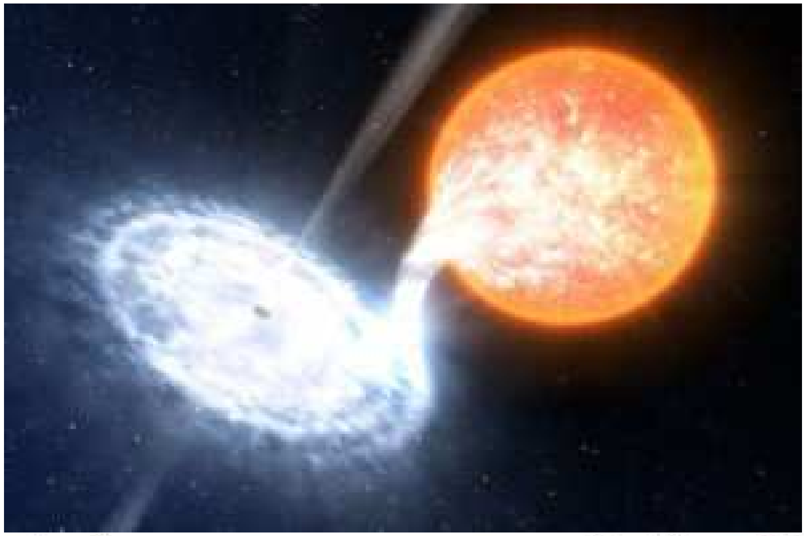


IX. Stadiul Stelelor Neutronice & Pulsarilor





X. Stadiul Găurilor Negre (Black Holes)



$$m^{\gamma}_{\mathbf{inertiala}}c^2 = h\upsilon_0 \Rightarrow m^{\gamma}_{\mathbf{inertiala}} = \frac{h\upsilon_0}{c^2}$$

$$m^{\gamma}_{\mathbf{inertiala}} = m^{\gamma}_{\mathbf{gravifica}}$$

$$U = -\frac{GMm^{\gamma}_{\mathbf{gravifica}}}{r} = -\frac{GMh}{rc^2}\upsilon_0$$

deplasare spre roșu gravitațională sau Einstein

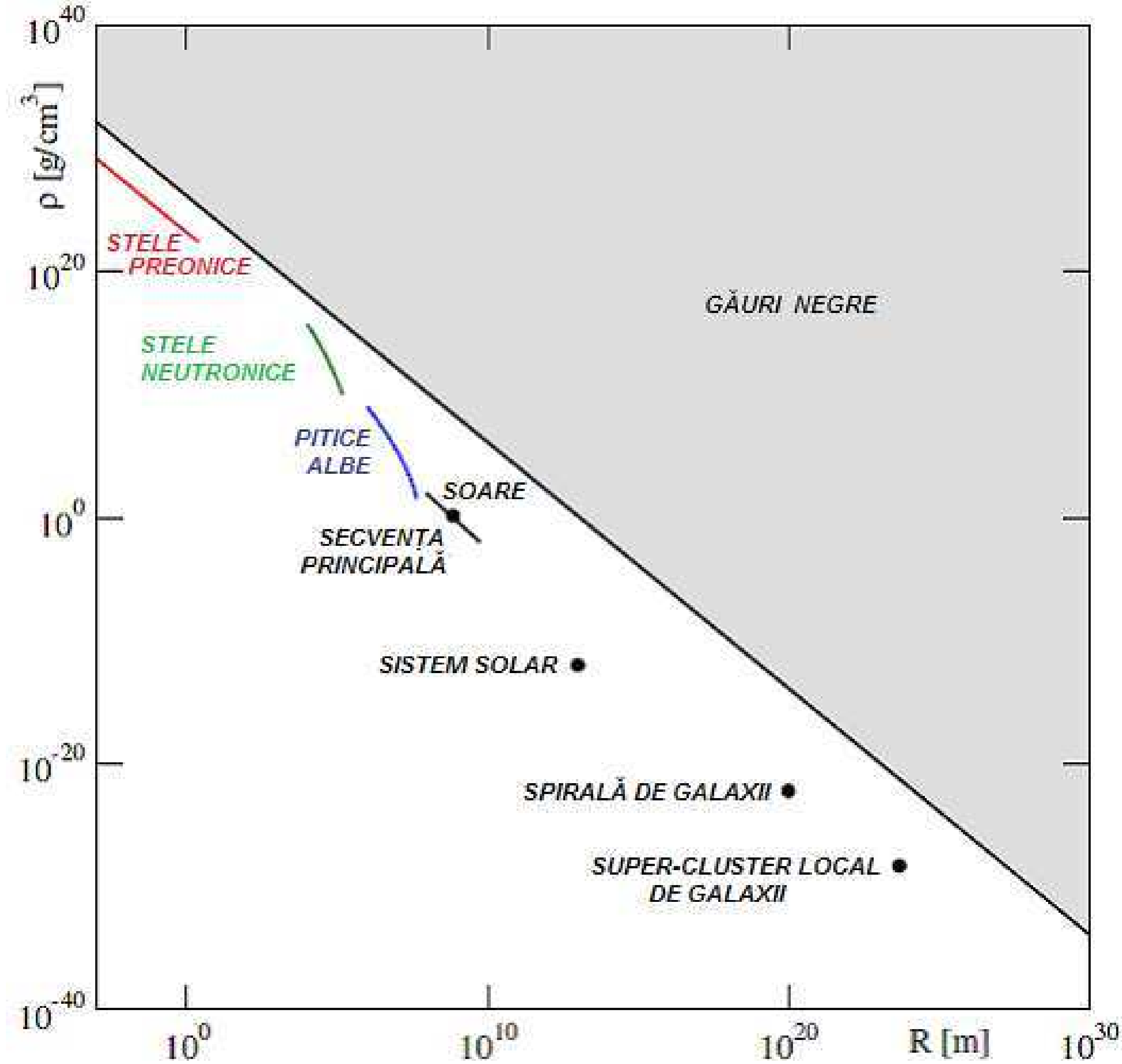
$$h\upsilon = \underbrace{h\upsilon_0}_{\substack{\text{miscare}\\\text{libera}}} \underbrace{-\frac{GMh}{rc^2}\upsilon_0}_{\substack{\text{miscare}\\\text{in}\\\text{camp}\\\text{gravitational}}} \Rightarrow \upsilon = \upsilon_0\left(1 - \frac{GM}{rc^2}\right)$$

$$h\upsilon_0 = \frac{GMh}{r_0c^2}\upsilon_0 \Rightarrow r_0 = \frac{GM}{c^2}$$

$$R_{Orizont} = 2r_0 = \frac{2GM}{c^2} = \begin{cases} 9km \ldots 3\times M_{Soare} \\ 3km \ldots 1\times M_{Soare} \\ 9mm \ldots 1\times M_{Pamant} \end{cases}$$

$$\frac{\Delta\upsilon}{\upsilon_0} = -\frac{GM}{rc^2}$$

$$\frac{mv^2}{2} = \frac{GMm}{r} \Rightarrow v_{evadare} = \sqrt{\frac{2GM}{r}} \overset{v_{evadare}=c}{\Rightarrow} r = R_{Orizont} = \frac{2GM}{c^2}$$


10^{40}
ρ [g/cm^3]
10^{20}
10^{0}
10^{-20}
10^{-40}
10^{0}
10^{10}
10^{20}
R [m]
10^{30}
STELE PREONICE
GĂURI NEGRE
STELE NEUTRONICE
PITICE ALBE
SOARE
SECVENȚA PRINCIPALĂ
SISTEM SOLAR
SPIRALĂ DE GALAXII
SUPER-CLUSTER LOCAL DE GALAXII

Modelul fermionic degenerat al piticelor albe.

Pentru fixarea cadrului de lucru reamintim condițiile fizico-chimice dintr-o stea albă pitică: *pentru o compoziție aproape exclusivă de heliu în stare de plasmă, se pune problema determinării masei acesteia, față de masa Soarelui, 10^{33} g, și relația în raport cu raza sa, știind că temperatura centrală a stelei este de cca $T_{Stea}=10^{7}$ K (precum în Soare), iar densitatea de electroni ρ_e este de cca 10^{30} electroni/cm^{3}, întro- comprimare extremă a stelei.*

Rezolvarea problemei se face în mai multe etape.

I. ***Stabilirea comportării electronice în steaua pitică albă.***

II. ***Explicitarea ecuației de stabilitate a stelei pitice albe***

III. ***Calculul presiunii gazului de electroni degenerat.***

IV. ***Discuția relațiilor masă-raza stelei degenerate.***

I. *Stabilirea comportării electronice în steaua pitică albă.*

$$\rho_e = \frac{N}{V} = \frac{1}{v}$$

$$\boxed{N = \frac{V}{v} = \frac{2V}{h^3}\left(\frac{4}{3}\pi p_F^3\right)}$$

$$\boxed{p_F = \hbar\left(\frac{3\pi^2}{v}\right)^{1/3}}$$

$$\varepsilon_F \cong \frac{p_F^2}{2m_e} \cong \frac{\hbar^2}{2m_e}\frac{1}{v^{2/3}} = \frac{\hbar^2}{2m_e}\rho_e^{2/3} \cong 20[MeV]$$

$$T_F = \frac{\varepsilon_F}{k_B} \cong 10^{11}[K] >> T_{Stea} = 10^7[K]$$

$$M = (m_e + 2m_p)N \cong 2m_p N \Rightarrow N \cong \frac{M}{2m_p}$$

$$v = \frac{V}{N} \cong \underbrace{\left(\frac{4}{3}\pi R^3\right)}_{V}\underbrace{\left(\frac{2m_p}{M}\right)}_{1/N} = \frac{8\pi}{3}m_p\frac{R^3}{M}$$

$$V = \frac{4}{3}\pi R^3$$

II. *Explicitarea ecuației de stabilitate a stelei pitice albe*

- Presiunea gazului de electroni apare ca reacție la existența unui lucru mecanic extern ce ar confina masa stelară de la starea de dilatare infinită la cea prezentă cu volumul de rază R; prin urmare are forma

- Self-energia gravitațională geenrează o energie pe baza interacțiunii mase stelare cu ea însăși

$$\int_{\infty}^{R} P_0 4\pi r^2 dr = -\frac{GM^2}{R}$$

$$P_0 4\pi R^2 = \frac{GM^2}{R^2} \Rightarrow \boxed{R^4 = \frac{1}{P_0}\frac{G}{4\pi}M^2}$$

III. *Calculul presiunii gazului de electroni degenerat*.

$$P_0 = -\frac{\partial E_0}{\partial V}$$

$$E_0(p) = \sum_{|\mathbf{p}|<|\mathbf{p}_F|} \sqrt{(cp)^2 + (m_e c^2)^2}$$

$$= \frac{2V}{h^3} \int_0^{p_F} dp 4\pi p^2 \sqrt{(cp)^2 + (m_e c^2)^2}$$

$$= \frac{2V}{h^3} 4\pi m_e^4 c^5 \int_0^{p_F/m_e c^2} d\left(\frac{p}{m_e c}\right)\left(\frac{p}{m_e c}\right)^2 \sqrt{1+\left(\frac{p}{m_e c}\right)^2}$$

$$= \frac{V}{\hbar^3 \pi^2} m_e^4 c^5 \int_0^{x_F} dx x^2 \sqrt{1+x^2}$$

$$\frac{E_0}{N} = \frac{m_e^4 c^5}{\hbar^3 \pi^2} v f(x_F)$$

$$f(x_F) = \int_0^{x_F} dx x^2 \sqrt{1+x^2} = \begin{cases} \frac{1}{3} x_F^3 \left(1 + \frac{3}{10} x_F^2 + \ldots\right), \ x_F << 1\, (\textbf{caz nerelativist}) \\ \frac{1}{4} x_F^3 \left(1 + \frac{1}{x_F^2} + \ldots\right), \ x_F >> 1\ (\textbf{caz relativist}) \end{cases}$$

$$\boxed{\overline{R} = \frac{m_e c}{\hbar} R}$$

$$\boxed{\overline{M} = \frac{9\pi}{8}\frac{M}{m_p}}$$

$$x_F = \frac{p_F}{m_e c} = \frac{\hbar}{m_e c}\left(\frac{3\pi^2}{v}\right)^{1/3}$$

$$= \frac{\hbar}{m_e c}\left(3\pi^2 \frac{3}{8\pi}\frac{M}{m_p}\frac{1}{R^3}\right)^{1/3} = \underbrace{\frac{\hbar}{m_e c}\frac{1}{R}}_{1/\overline{R}}\underbrace{\left(\frac{9\pi}{8}\frac{M}{m_p}\right)^{1/3}}_{\overline{M}} = \frac{\overline{M}^{1/3}}{\overline{R}}$$

$$P_0 = -\frac{\partial (E_0 / N)}{\partial v} = \frac{m_e^4 c^5}{\hbar^3 \pi^2}\left[-f(x_F) - \frac{\partial f(x_F)}{\partial x_F} v \frac{\partial x_F}{\partial v}\right] = \frac{m_e^4 c^5}{\hbar^3 \pi^2}\left[\frac{1}{3} x_F^3 \sqrt{1+x_F^2} - f(x_F)\right]$$

$$= \begin{cases} \frac{1}{15}\frac{m_e^4 c^5}{\hbar^3 \pi^2} x_F^5 \ldots x_F \ll 1 \,(\textbf{caz nerelativist}) \\ \frac{1}{12}\frac{m_e^4 c^5}{\hbar^3 \pi^2}\left(x_F^4 - x_F^2\right) \ldots x_F \gg 1 \,(\textbf{caz relativist}) \end{cases}$$

$$= \begin{cases} \frac{4}{5} K \frac{\overline{M}^{5/3}}{\overline{R}^5} \ldots \textbf{caz nerelativist} \\ K\left(\frac{\overline{M}^{4/3}}{\overline{R}^4} - \frac{\overline{M}^{2/3}}{\overline{R}^2}\right) \ldots \textbf{caz relativist} \end{cases}$$

$$K = \frac{m_e c^2}{12\pi^2}\left(\frac{m_e c}{\hbar}\right)^3$$

IV. *Discuția relațiilor masă-raza stelei degenerate.*

$$P_0 = \frac{G}{4\pi}\frac{M^2}{R^4} = \underbrace{\frac{G}{4\pi}\left(\frac{8m_p}{9\pi}\right)^2\left(\frac{m_e c}{\hbar}\right)^4}_{\equiv K^*}\frac{\overline{M}^2}{\overline{R}^4} = K^*\frac{\overline{M}^2}{\overline{R}^4}$$

$$K^* = \frac{G}{4\pi}\left(\frac{8m_p}{9\pi}\right)^2\left(\frac{m_e c}{\hbar}\right)^4$$

A. Pentru cazul nerelativist

$$\overline{R} = \frac{4}{5}\frac{K}{K^*}\frac{1}{\overline{M}^{1/3}}$$

$$\frac{K}{K^*} = \overline{M}_0^{2/3}$$

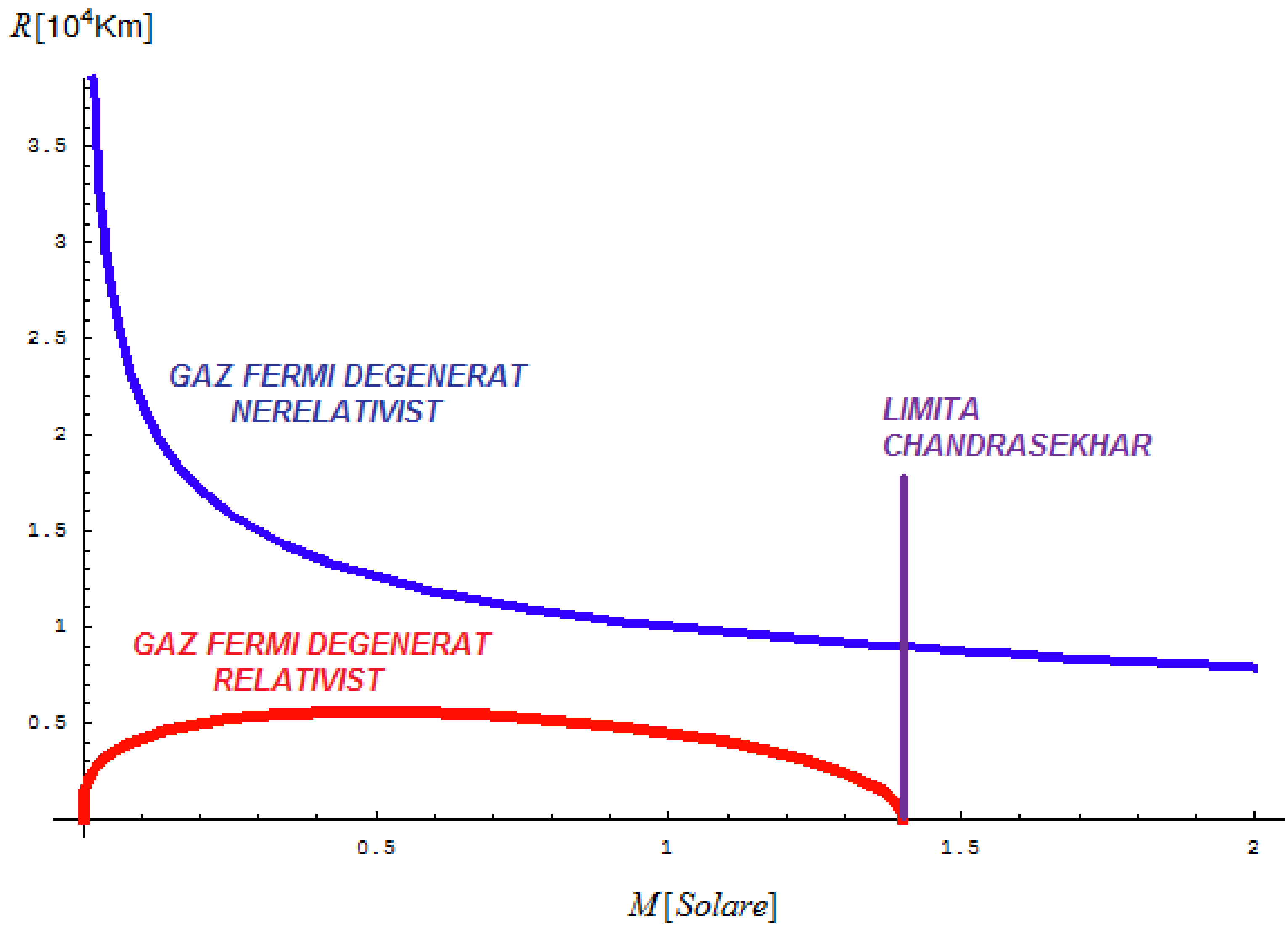
$$\boxed{\overline{R}_{nerelativist} = \frac{4}{5}\frac{\overline{M}_0^{2/3}}{\overline{M}^{1/3}}}$$

B. Pentru cazul relativist

$$\boxed{\overline{R}_{relativist} = \overline{M}^{1/3}\sqrt{1-\left(\frac{\overline{M}}{\overline{M}_0}\right)^{2/3}}}$$

$$\overline{M}_0 = \left(\frac{K}{K^*}\right)^{3/2} = \left(\frac{27\pi}{64}\right)^{3/2}\underbrace{\left(\frac{\hbar c}{G m_p^2}\right)^{3/2}}_{\cong 10^{39}}$$

$$M_0 = \frac{8}{9\pi}m_p\overline{M}_0 \cong 10^{33}\,g \sim M_{Soare}$$


R[10^4Km]
3.5
3
2.5
2
1.5
1
0.5
GAZ FERMI DEGENERAT
NERELATIVIST
LIMITA
CHANDRASEKHAR
GAZ FERMI DEGENERAT
RELATIVIST
0.5
1
1.5
2
M[Solare]

Subrahmanyan Chandrasekhar

Sir Fred Hoyle