Lecture 11: Quantum Postulates

I. The Wave-Function Continuity

Each particle, either in free movement or in bound state under a potential that do not depend explicitly on time may be associated with a wave-function ψ which contains, in principle all, information of the system; moreover:

- it is a continuously function of coordinate (including for classical turning points);

$$\lim_{x>a}\psi(x)=\lim_{x<a}\psi(x)\ ,\forall a\in\Re$$

- it is at least one fold derivable respecting coordinate(s) and with it as a continuous function of coordinate(s) as well;

$$\lim_{x>a}\partial_x\psi(x)=\lim_{x<a}\partial_x\psi(x),\forall a\in\Re$$

II. Eigen-energies and Eigen-functions

An observable system is represented by stationary wave-function $\psi(x)$ satisfying the analytical constraints:

- is a solution of the stationary Schrödinger equation

$$\hat{H}\psi=E\psi$$

- is normalizable, i.e. asymptotically vanishes when $x\to\infty$;

$$\int\psi^*\psi d\Gamma=1,$$

$$\lim_{x\to\pm\infty}\psi(x)=\lim_{x\to\pm\infty}\psi^*(x)=0$$

This principle goes together with the fact that all functions that are solution to a given Schrödinger equation may be linearly combined (superimposed) in producing other proper (eigen) wave-functions of the system;

$$\exists\varphi_i:\hat{H}\varphi_i=E\varphi_i\Rightarrow\exists\psi=\sum_i c_i\varphi_i, c_i\in\Im\ \Big|\ \hat{H}\psi=E\psi$$

III. Variational Wave-function and Energy

The stationary Schrödinger equation may be integrated to its eigen-values by means of the variational principle respecting the minimizing of the total (eigen) energy of the system:

$$\delta E = 0\,, \quad E = \frac{\int \psi^{*} \hat{H} \psi d\Gamma}{\int \psi^{*} \psi d\Gamma}$$

with ψ being a trial stationary wave-function suitable for the concerned system, in accordance with the previous enounced quantum postulates.

With this principle, actually all, natural systems may be quantum mechanically treated with the aid of trial-and-optimized wave-function to determine the correspondent eigen-energies. In next some of the most representative systems are to be accordingly treated from nuclear, to atomic, to molecular, and to solid state level or matter's organization.